EXPLANATION OF THE TOMS EFFECT IN TERMS OF THE VISCOSITY ANISOTROPY OF THE SOLUTION

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In [1] Toms experimentally established that in a certain concentration range the drag of a turbulent tube flow of a solution of polymethyl methacrylate in monochlorobenzene is less than for the solvent. Subsequently, this effect was observed in connection with the turbulent flow of dilute aqueous solutions in tubes, between rotating coaxial cylinders, and also in flow around streamlined and bluff bodies. High-molecular compounds of both biological and synthetic origin have been used as additives [2–7].

Numerous investigations have revealed the following properties of flows of solutions that exhibit the Toms effect. First, the effect is substantial only when linear polymers are used, and branching has an adverse influence [6]. The important role of the elongation of the particles has been rigorously demonstrated in experiments on tube flows of water containing nylon fibers with elongations of from 11.69 to 51.14 [8]. Secondly, the dissolved particles chiefly affect a layer of fluid near the walls, without causing changes in the zone of developed turbulent flow [9–11].

In [12], in order to explain the Toms effect, a hypothesis was proposed that takes into account these two properties of solution flows. According to this hypothesis, as a result of the large velocity gradient in the viscous sublayer and transition layer the elongated particles are oriented along the flow, thus damping the transverse velocity fluctuations v'. This leads to a reduction of the turbulent friction $\rho_0\langle u'v'\rangle$, where u' is the fluctuation of the longitudinal velocity component.

In what follows qualitative calculations based on the proposed hypothesis are compared with the experimental data.

The orientation of elongated particles in a laminar flow due to the velocity gradient has been demonstrated theoretically (for example, [13], pp. 499–521). The experimental confirmation follows from an analysis of the curves representing the friction stress in the solution τ as a function of velocity gradient ([12, 13], ch. II) and the birefringence in the flow ([13], ch. VII, VIII). The viscosity of the solution decreases with increase in velocity gradient (Fig. 1).

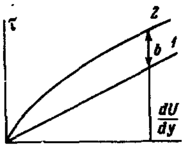


Fig. 1. 1) Newtonian fluid (solvent), 2) solution.

We denote by μ_x the coefficient of viscosity of the solution for flow in the direction of the x-axis leading to orientation of the elongated particles (Fig. 2a). In highly dilute solutions

$$\mu_x = \mu_0 (1 + c[\mu]). \tag{1}$$

Here μ_0 is the viscosity of the solvent, c is the concentration, and $[\mu]$ is the intrinsic viscosity, which depends on the velocity gradient.

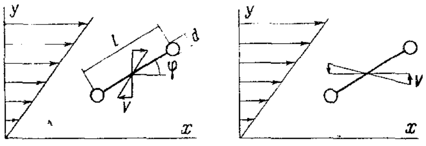
If in a flow of solution with an angle of most probable orientation φ_m a secondary flow with shear along the y-axis (Fig. 2b) develops, for example, as a result of turbulent velocity fluctuations, we denote the corresponding viscosity coefficient by μ_y. At sufficiently small velocity gradients, when $\varphi_m = 45°$, $\mu_x = \mu_y$. With increase in the velocity gradient of the flow along the x-axis the angle $\varphi_m \to 0$, as a result of which μ_x decreases, while μ_y increases. We call the ratio

$$K_a = \mu_y/\mu_x$$

the dynamic viscosity anisotropy of the solution.

At sufficiently small velocity gradients, $K_a = 1$; at large gradients it is always greater than unity. For any gradient it can be calculated, since the quantity μ_y is also expressed in terms of the intrinsic viscosity by Eq. (1), but in this case the intrinsic viscosity should correspond to shear along the y-axis.

Let us consider the behavior of the fluid in the viscous sublayer and the transition layer of the turbulent flow, where the velocity gradient along the x-axis is large (greater than 10^2). Accordingly, it may be assumed that $\varphi_m \approx 0$, and then the calculation of K_a is considerably simplified.



a b

Fig. 2.

We will demonstrate this with reference to the example of a solution containing simple dumbbell-shaped elongated particles. A calculation method for particles of more complicated configuration is described in [14].

The dissipation of flow energy by the particle, leading to an increase in the viscosity of the solution as compared with the solvent, depends linearly on the flow velocity V (Fig. 2b):

$$V = {}^1/_2\, l \cos\varphi\, g \tag{2}$$

Here, l is the length of the dumbbell, φ is the orientation angle, and g is the velocity gradient.

For $\varphi = 0$ and shear along the x-axis $V_1 = dg/2$ (here, d is the diameter of the ball of the dumbbell); for $\varphi = 0$ and shear along the y-axis the Stokes velocity

$$V_2 = {}^1/_2 lg$$

Hence it is clear that for a highly elongated particle ($l/d \gg 1$)

$$\mu_y > \mu_x$$

As the elongation of the particle increases, so does the value of K_a.

In solution macromolecules with a high molecular weight have a configuration more complicated than that of dumbbells, rods, ellipsoids, and similar simple shapes. However, the proposed approach is applicable to them also, as follows from the experimental determination of the variation of intrinsic viscosity with velocity gradient. Thus, for example, in a solution of poly-γ-benzyl L-glutamate with a molecular weight of $1.3 \cdot 10^5$ the intrinsic viscosity decreases by a factor of approximately 7 with increase in velocity gradient, and by a factor of approximately 20 at a molecular weight of $3.36 \cdot 10^5$. When the same molecules pass from a cylindrical spiral conformation to a statistical coil there is no decrease in intrinsic viscosity ([13], p. 174.)

For purposes of the subsequent qualitative calculations we will determine the limiting value of K_a characterizing a dilute solution of highly elongated particles with a small transverse dimension in the presence of a high velocity gradient, i.e., when $\varphi_m = 0$. Under these conditions $\mu_x \approx \mu_0$ (Fig. 1, [12]), and μ_y is easily calculated using the value of the intrinsic viscosity at very small velocity gradients. Since at small velocity gradients $\varphi_m = 45°$, whereas at large gradients $\varphi_m = 0$, the ratio of the velocities at these two orientation angles, when the shear is along the y-axis, is equal to cos 45° (Eq. (2)). In the presence of Stokes flow the drag is directly proportional to the freestream velocity; therefore the ratio of the intrinsic viscosities is also equal to cos 45°. Consequently,

$$\mu_y = \mu_0\left(1 + \frac{c\,[\mu]_0}{\cos 45°}\right)$$

where $[\mu]_0$ is the intrinsic viscosity at very small velocity gradients. For the limiting value of the dynamic viscosity anisotropy we obtain

$$K_a = 1 + \sqrt{2c}\,[\mu]_0 . \qquad (3)$$

All this relates to laminar flows. However, the principal requirement—the presence of a large velocity gradient in one direction—is also satisfied in the turbulent boundary layer (in the viscous sublayer and the transition layer). In this zone the elongated particles should be oriented with the long axis along the wall. The fluctuations in the viscous sublayer and transition layer create fluctuational velocity gradients.

The dissipation of fluctuating motion along and across the flow is determined by the longitudinal μ_x and transverse μ_y viscosities. As a result of the dynamic viscosity anisotropy the transverse fluctuations should be more strongly damped. In the turbulent core, where the averaged velocity gradient is relatively small, there is practically no dynamic viscosity anisotropy, and the flow behaves like a Newtonian fluid.

This property of solution flow has been experimentally confirmed by Khabakhpasheva (Siberian Thermophysics Colloquium, 1968; for a description of the measurements see [15]). In that part of the viscous sublayer and the transition layer where it has proved possible to obtain experimental data, the transverse velocity fluctuations in an aqueous solution were found to be approximately three times less than in water, and the longitudinal fluctuations in a certain zone even greater.

In order to calculate the Toms effect we will start from the experimentally established relations for the quasi-stationary thickness of the viscous sublayer and the averaged velocity distribution in the sublayer for a Newtonian fluid flow:

$$\delta_0 = a\frac{\nu_0}{\sqrt{\tau_w/\rho_0}} \qquad \left(\nu_0 = \frac{\mu_0}{\rho_0}\right) \qquad (4)$$

$$U = \frac{\tau_w}{\mu_0}\,y \quad \text{and} \quad y \leqslant \delta_0 \qquad (5)$$

Here, δ_0 is the thickness of the viscous sublayer, ν_0 is the kinematic viscosity, τ_w is the friction stress at the wall, ρ_0 is the solvent density, a is an experimentally determined constant, and U is the averaged longitudinal velocity.

Let us compare the flows of solvent (Newtonian fluid) and solution at the same averaged velocity gradient in the viscous sublayer. The dependence of the friction stress at the wall on the averaged velocity gradient in the solution is represented in a form analogous to (5):

$$\tau_w = \mu_x\frac{dU}{dy} + b \qquad (6)$$

Here, b is the non-Newtonian correction, a function of the velocity gradient whose significance is clear from Fig. 1. Since as

$$\frac{dU}{dy} \to \infty, \quad \mu_x \to \mu_0 \text{ and } \frac{b}{\mu_x\, dU/dy} \to 0$$

for a sufficiently large averaged velocity gradient we can write

$$\tau_w \approx \bar{\tau}_w{}^* \qquad (7)$$

where $\tau_\omega{}^*$ is the friction stress at the wall in the solution.

Since in the solvent and solution flows only the two viscosities μ_y and μ_0 differ, for the thickness of the viscous sublayer in the solution we can write the general expression

$$\delta_0{}^* = a_1\frac{\nu_0 f(\nu_y/\nu_0)}{\sqrt{\tau_w{}^*/\rho}} \qquad (8)$$

Here a_1 is a constant, ρ is the density of the solution, and $f(\nu_y/\nu_0)$ is a still unknown function.

The form of the function f can be found by considering the action of the velocity fluctuations on the particles oriented by the averaged flow. The simplest form of this function is

$$f\left(\frac{\nu_y}{\nu_0}\right) = \frac{\nu_y}{\nu_0}$$

Moreover, we assume that $a = a_1$. If we now use relation (7) and the approximation $\rho \approx \rho_0$, which is valid for dilute solutions, then for the ratio of the thicknesses of the viscous sublayer we can write:

$$\frac{\delta_0^*}{\delta_0} \approx \frac{\mu_y}{\mu_0} = K_a \tag{9}$$

i.e., the thickness of the viscous sublayer in the solution is greater than that in the solvent by a factor of approximately K_a.

We now compare the flows of solvent and solution at the same averaged velocity gradients in the viscous sublayer. Accordingly, at the outer edge of the viscous sublayer in the solution the averaged velocity will be greater than in the solvent by ΔU (using relations (1), (2), and (7)):

$$\Delta U = U_l^* - U_l \approx \frac{\tau_w}{\mu_0}(\delta_0^* - \delta_0) \approx \frac{\tau_w \delta_0}{\mu_0}(K_a - 1) \tag{10}$$

where U_l is the averaged velocity at the outer edge of the viscous sublayer.

For a qualitative calculation the constant a may be taken equal to 10 [16].

As a result, instead of (10) we obtain

$$\Delta U \approx 10 \sqrt{\tau_w / \rho_0}\,(K_a - 1) \tag{11}$$

The velocity increment ΔU, obtained at the outer edge of the viscous sublayer (in this qualitative calculation the transition layer is disregarded), will be the velocity increment at the outer edge of the boundary layer, since in the turbulent core the solution behaves like a Newtonian fluid. In order to relate the local coefficient of friction at the wall with the velocity at the outer edge of the boundary layer we employ the following formula, valid at Reynolds numbers R_x of the order of 10^7 (for another range of Reynolds numbers it is necessary to take the appropriate relation from [17]):

$$c_f = 0.0576\,(R_x)^{-1/5} \qquad \left(R_x = \frac{U_0 x}{\nu_0}\,,\ c_f = \frac{2\tau_w}{\rho_0 U_0^2}\right) \tag{12}$$

Here, c_f is the local coefficient of friction; U_0 is the velocity at the outer edge of the boundary layer. Then

$$\frac{\Delta U}{U_0} \approx \frac{10\sqrt{0.0576}\,(K_a - 1)}{\sqrt{2}\,(R_x)^{0.1}} \tag{13}$$

Using (3), we finally obtain

$$\frac{\Delta U}{U_0} \approx \frac{5\sqrt{0.0576}\,[\mu]_0\, c}{(R_x)^{0.1}} \tag{14}$$

Qualitative expressions have now been obtained for the relation between the Toms effect and the characteristics of the boundary layer and the solution. If quantitative data are required, it will be necessary, first, to take into account the finiteness of the particle elongation; we assumed that the elongation $\gg 1$. In this case it may turn out that μ_x is appreciably greater than μ_0. It should also be kept in mind that the elongation of a deformable particle depends on the shear stress. Secondly, finding the form of the function $f(\nu_y/\nu_0)$ (8) requires theoretical and experimental investigations of the dependence of the thickness of the viscous sublayer of the solution on the intensity of the transverse velocity fluctuations in the viscous sublayer and the transition layer.

The final expression (14) not only correctly reflects the laws of turbulent flow of solutions of chain macromolecules but also gives quantitative results consistent with the experimental data. Thus, for example, at a solution viscosity $\mu = 2$ cP and Reynolds number $R_x = 10^7$, Eq. (15) gives an increase in velocity by a factor of 2. This is in good agreement with the experimental data of [1–7], which indicate a velocity increment of 30–50%.

It remains to mention the dependence of the Toms effect on the concentration of the solution (Eq. (14)): this becomes more pronounced as the concentration increases. It has been experimentally established that any polymer has an optimum concentration at which the Toms effect is maximum, diminishing as the concentration continues to increase. However, this does not contradict relation (14), since it was obtained for very dilute solutions and thus characterizes only part of the dependence of $U^{-1}_0\Delta U$ on c.

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